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A Multi-Echelon Supply Chain Inventory Model With Variable Demand Rate And Variable Deterioration

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ABSTRACT:
In this paper**,** an integrated supply chain model has been developed for varying demand of deteriorating items with life time. The variable rate of deterioration has been taken. In this model, three suppliers, one distributor and three retailers are considered. The replenishment of items at the distributor and retailers is considered as instantaneous. Shortage of items is not allowed. Cost minimization technique has been used to obtain the optimal values of the parameter.
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KEYWORDS: Deterioration, supply chain, three- echelon inventory system.

INTRODUCTION:
In recent year the interest of researchers has been increased in a study of supply chain management. A supply chain is a network of suppliers, manufacturers, distributors and retailers that perform the function of procurement of material, transformation of these materials into intermediate and finished product and distribution of this product to customers. This chain is traditionally characterized by forward flow of materials and backward flow of information. The three key members of the supply chain (supplier, distributor and the retailer) managed independently their inventories. Increasing competitive pressure and due to decrease in margin of profit the company force to develop supply chain which provide quick response to customers' needs and reducing the cost of carrying material. The number of deliveries is derived in cooperation with each other to achieve a minimum total integrated cost.
Andrew J. Clark and Herbert Scarf [2] in year 1960 were the first to consider the echelon stock in the inventory system. S.A Besseler and A. J Veinote [1] in year 1966 discussed the optimal policy for a multi-echelon inventory model.
H.M Wee [18] in year1998 discussed the optimal-buyer- seller discounting pricing and ordering policy for deteriorating items. Risk reduction of items shortage is also an important factor in supply management. The shortage of items may be avoided through suppliers in a supply chain. In earlier literature, it is assumed that inventory is replenished by a single supplier. But in reality there are situations when more than one supplier is required to satisfy a desirable goal. Dayani Segarage,O.Fujiwawara and Huynh Trang Luong [16] in year1999 presented mathematical model of N-supplier inventory systems and derived optimal inventory policy. Ranga V. Ramasesh , J. Keith Ord and Jack C. Hayya [14] in year 1993 showed that dual sourcing is required when requirement is large or lead time is uncertain. Hon Shiang Lau and Long Geng Zha [11] in year 1993 presented optimal ordering policies with two suppliers when lead times and demand are all stochastic.
Deterioration of items is also an important factor in the development of inventory system. Deteriorating items refers to the items that become decayed, damaged, evaporative, expired, devaluation and so on through time. The inventory problem of deteriorating items was first studied by Whithin [19] in year 1957.Then P.M Ghare and G.F Schrader [3] in year 1963 developed the model for exponential decaying inventory. F. Raafat

[13] in year 1991 and S.K Goyal and B.C Giri [4] in year 2001 made a comprehensive literature review on deteriorating inventory items.

K. Sawaki [15] in year 2003 developed optimal policies in continuous time inventory models with limited supply. P.N Mishra and Manish Goyal [12] in year 2016 developed the model of two level optimal ordering policies for variable deterioration with life time under trended demand and time discounting. Yu [20] in year 2007 developed a mathematical model of deteriorating items by considering a vertical integration of producer, the distributor and the retailer as well a horizontal integration of the suppliers. The model presented by them has potential application in a multi- echelon supply chain inventory system.

In this paper, the work of Yu has been extended by considering linear demand rate and variable rate of deterioration. The replenishment of items as the distributor and retailers is considered as instantaneous. Shortage of items is not allowed. Cost Minimization technique has been used to obtain the optimal values of the parameter.

ASSUMPTIONS AND NOTATIONS:

ASSUMPTION:

The following assumptions are considered in this paper

(i) A single –product item is considered.

(ii) The demand rate is linear function of time.

(iii) No shortage is allowed.

(iv) The rate of replenishment at the distributor and the retailers is instantaneous.

(v) There is no constraint in space or capital.

(vi) A constant fraction of on hand inventory deteriorates after a specific time μ (the life time) and no replacement of deteriorated items is allowed. The variable rate of deterioration has been taken.

NOTATIONS:

The following notations have been used in this paper

TC : The joint cost per unit time for the supply chain.

TC_r : The total cost per unit time for the retailers.

TC_d : The total cost per unit time for the distributor.

TC_s : The total cost per unit time for the suppliers.

$I_{rk}(t)$: Inventory level of retailer *k* at time *t*.

I_d (t) :Inventory level of distributor at time *t*.

$I_{st}(t)$: Inventory level of supplier i at time t where *i*=1, 2

C_{rk} : The ordering cost for retailer k.

C_d : The ordering cost for the distributor.

$C_{s\,i}$: The ordering cost for supplier i.

H_{rk} : The carrying cost per unit per unit time for retailer *k*, where *k*=1, 2, 3.

H_d : The carrying cost unit per unit time for distributor

H_{si} : The carrying cost per unit per unit time for supplier i, where i=1, 2.

D_{rk} : The deterioration cost per unit per unit time for retailer *k* where *k*=1, 2, 3.

D_d : The deterioration cost per unit per unit time for distributor.

D_{si} : The deterioration cost per unit per unit time for supplier *i*. *i*=1, 2.

$\theta_o t$: The variable deterioration rate of inventory.

μ : The life time after which the deterioration of items starts.

T : The supply cycle time for the producer.

t_{rk} : The replenishment cycle time for retailer *k*.

t_d: The replenishment cycle time for distributor.

n_{sd} : The number of deliveries from the suppliers to the distributor during time *T*.

n_{dr} : The number of deliveries from the distributor to the retailers during time T

MATHEMATICAL MODELS AND ANALYSIS

The *N*-supplier- one-distributor-*N*-retailers inventory system can be represented as in fig.1; fig.2 and fig. 3 .We consider the supply chain system consisting of three suppliers, one distributor and three retailers. Now we develop the cost structures of suppliers, distributor and the retailers as follows:

(a) Cost Structure of *N*-retailers:

We consider the number of retailers *N* equal to three in the supply chain. Let $I_{rk}(t)$be the retailer's inventory level at any time t and demand rate be linear function given by a_kt + b_k . Let the inventory starts to deteriorate after the life time μ at the rate of θ_ot .The inventory system at the retailer is given in figure-1 given below:

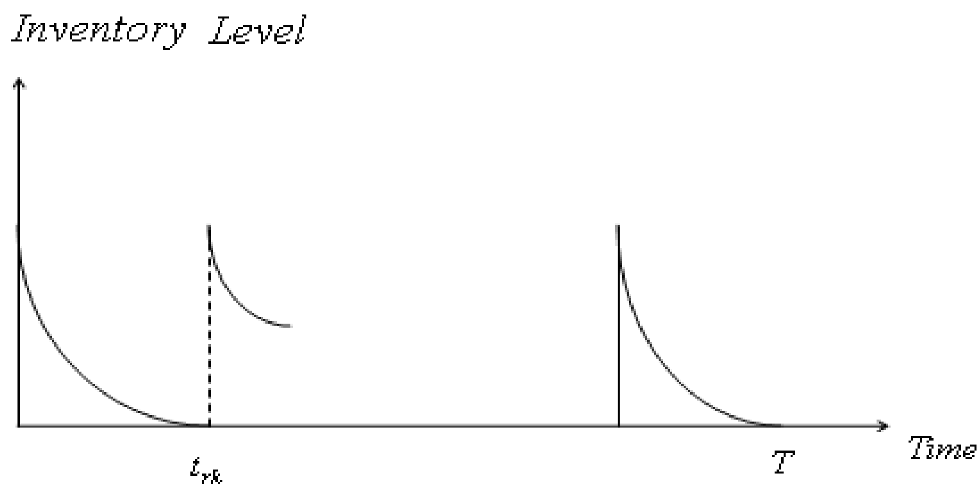

Figure-1

The differential equation governing the retailer's inventory level is given by

$$\frac{dI_{rk}(t)}{dt} = -(a_k t + b_k) \quad , \qquad 0 \le t \le \mu \qquad (1)$$

$$\frac{dI_{rk}(t)}{dt} = -(a_k t + b_k) - \theta_o t\, I_{rk} \, , \qquad \mu \le t \le t_{rk} \qquad (2)$$

The various boundary conditions are given by

$$I_{rk}(0) = S_{rk} \, , \; I_{rk}(t_{rk}) = 0 \qquad (3)$$

The solution of equation (1) is given by

$$I_{rk}(t) = -\left(\frac{a_k t^2}{2} + b_k t\right) + c_1 \qquad (4)$$

Using boundary condition (3) in equation (4), we get

$$c_1 = S_{rk}$$

Now substituting the value of c_1 in equation (4) , we get

$$I_{rk}(t) = -\left(\frac{a_k t^2}{2} + b_k t\right) + S_{rk} \quad , \quad 0 \le t \le \mu \qquad (5)$$

The solution of equation (2) is given by

$$I_{rk}(t) = -\left(a_k \frac{t^2}{2} + b_k t + a_k \frac{t^4 \theta_o}{8} + b_k \frac{t^3 \theta_o}{6}\right) e^{-\frac{\theta_o t^2}{2}} + c_2 e^{-\frac{\theta_o t^2}{2}} \qquad (6)$$

Using boundary condition (3) in equation (6) , we get

$$I_{rk}(t) = \left[\frac{a_k}{2}(t^2{}_{rk} - t^2) + b_k(t_{rk} - t) + \frac{a_k\theta_o}{8}(t^4{}_{rk} - t^4) + \frac{b_k\theta_o}{6}(t^3{}_{rk} - t^3)\right] e^{-\frac{\theta_o t^2}{2}}, \quad (7)$$

Where, $\mu \le t \le t_{rk}$

From (5) and (7), we have

$$-\left(\frac{a_k \mu^2}{2} + b_k\mu\right) + S_{rk} = \left[\frac{a_k}{2}(t^2{}_{rk} - \mu^2) + b_k(t_{rk} - \mu) + \frac{a_k\theta_o}{8}(t^4{}_{rk} - \mu^4) + \frac{b_k\theta_o}{6}(t^3{}_{rk} - \mu^3)\right] e^{-\frac{\theta_o \mu^2}{2}}$$

$$\therefore S_{rk} = \left(\frac{a_k \mu^2}{2} + b_k\mu\right) + \left[\frac{a_k}{2}(t^2{}_{rk} - \mu^2) + b_k(t_{rk} - \mu) + \frac{a_k\theta_o}{8}(t^4{}_{rk} - \mu^4) + \frac{b_k\theta_o}{6}(t^3{}_{rk} - \mu^3)\right] e^{-\frac{\theta_o \mu^2}{2}}$$

$$= a_k\left[\frac{\mu^2}{2} + \left\{\frac{1}{2}(t^2{}_{rk} - \mu^2) + \frac{\theta_o}{8}(t^4{}_{rk} - \mu^4)\right\} e^{-\frac{\theta_o \mu^2}{2}}\right] + b_k\left[\mu + \left\{(t_{rk} - \mu) + \frac{\theta_o}{6}(t^3{}_{rk} - \mu^3)\right\} e^{-\frac{\theta_o \mu^2}{2}}\right] \quad (8)$$

The deteriorating quantities per replenishment cycle can be obtained as

$$R_{rk} = S_{rk} - \int_0^{t_{rk}} (a_k t + b_k)\, dt$$

$$= S_{rk} - \left(a_k \frac{t^2{}_{rk}}{2} + b_k t_{rk}\right)$$

$$= a_k\left[\frac{\mu^2}{2} + \left\{\frac{1}{2}(t^2{}_{rk} - \mu^2) + \frac{\theta_o}{8}(t^4{}_{rk} - \mu^4)\right\} e^{-\frac{\theta_o \mu^2}{2}}\right] + b_k\left[\mu + \left\{(t_{rk} - \mu) + \frac{\theta_o}{6}(t^3{}_{rk} - \mu^3)\right\} e^{-\frac{\theta_o \mu^2}{2}}\right] - \left(a_k \frac{t^2{}_{rk}}{2} + b_k t_{rk}\right) = A_{rk}\theta_o t \quad (9)$$

The deterioration cost per replenishment cycle is given by

$$DC_{rk} = R_{rk} \times D_{rk}$$

$$= [\, a_k\left[\frac{\mu^2}{2} + \left\{\frac{1}{2}(t^2{}_{rk} - \mu^2) + \frac{\theta_o}{8}(t^4{}_{rk} - \mu^4)\right\} e^{-\frac{\theta_o \mu^2}{2}}\right] + b_k\left[\mu + \left\{(t_{rk} - \mu) + \frac{\theta_o}{6}(t^3{}_{rk} - \mu^3)\right\} e^{-\frac{\theta_o \mu^2}{2}}\right] - \left(a_k \frac{t^2{}_{rk}}{2} + b_k t_{rk}\right)] \times D_{rk} \quad (10)$$

From equation (9), we have

$$A_{rk} = \frac{a_k}{\theta_o t}\left[\frac{\mu^2}{2} + \left\{\frac{1}{2}(t^2{}_{rk} - \mu^2) + \frac{\theta_o}{8}(t^4{}_{rk} - \mu^4)\right\} e^{-\frac{\theta_o \mu^2}{2}}\right] + \frac{b_k}{\theta_o t}\left[\mu + \left\{(t_{rk} - \mu) + \frac{\theta_o}{6}(t^3{}_{rk} - \mu^3)\right\} e^{-\frac{\theta_o \mu^2}{2}}\right] - \frac{1}{\theta_o t}\left(a_k \frac{t^2{}_{rk}}{2} + b_k t_{rk}\right) \quad (11)$$

The inventory carrying cost per replenishment cycle is given by

$$HC_{rk} = A_{rk} \times H_{rk}$$

$$= [\, \frac{a_k}{\theta_o t}\left[\frac{\mu^2}{2} + \left\{\frac{1}{2}(t^2{}_{rk} - \mu^2) + \frac{\theta_o}{8}(t^4{}_{rk} - \mu^4)\right\} e^{-\frac{\theta_o \mu^2}{2}}\right] + \frac{b_k}{\theta_o t}\left[\mu + \left\{(t_{rk} - \mu) + \frac{\theta_o}{6}(t^3{}_{rk} - \mu^3)\right\} e^{-\frac{\theta_o \mu^2}{2}}\right] - \frac{1}{\theta_o t}\left(a_k \frac{t^2{}_{rk}}{2} + b_k t_{rk}\right)\,]\, H_{rk} \quad (12)$$

The retailer's total cost per replenishment cycle can be expressed as the sum of ordering cost transportation cost, the carrying cost and the deterioration cost. Thus we have

$$TC_r(t_{rk}) = \sum_{k=1}^{3}\Bigg[C_{rk} + \left[\frac{a_k}{\theta_o t}\left[\frac{\mu^2}{2} + \left\{\frac{1}{2}(t^2{}_{rk} - \mu^2) + \frac{\theta_o}{8}(t^4{}_{rk} - \mu^4)\right\} e^{-\frac{\theta_o \mu^2}{2}}\right] + \frac{b_k}{\theta_o t}\left[\mu + \left\{(t_{rk} - \mu) + \frac{\theta_o}{6}(t^3{}_{rk} - \mu^3)\right\} e^{-\frac{\theta_o \mu^2}{2}}\right] - \frac{1}{\theta_o t}\left(a_k \frac{t^2{}_{rk}}{2} + b_k t_{rk}\right)\right] H_{rk} + a_k\left[\frac{\mu^2}{2} + \left\{\frac{1}{2}(t^2{}_{rk} - \mu^2) + \frac{\theta_o}{8}(t^4{}_{rk} -\right.\right.$$

$\mu^4)\Big\} e^{-\frac{\theta_o \mu^2}{2}}\Big] + b_k \Big[\mu + \Big\{(t_{rk} - \mu) + \frac{\theta_o}{6}\ (t^3{}_{rk} - \mu^3)\Big\} e^{-\frac{\theta_o \mu^2}{2}}\Big] - \Big(a_k \frac{t^2{}_{rk}}{2} + b_k t_{rk}\Big)]D_{rk} + (a +$
$bQ_r)$ $\Big]$ (13)

The retailer's total cost during cycle T is given by

$TC_r(t) = (n_{sd} n_{dr}) \sum_{k=1}^{3} [\, C_{rk} + \Big[\frac{a_k}{\theta_o t}\Big[\frac{\mu^2}{2} + \Big\{\frac{1}{2}\Big(\frac{T^2}{(n_{sd} n_{dr})^2} - \mu^2\Big) + \frac{\theta_o}{8}\Big(\frac{T^4}{(n_{sd} n_{dr})^4} - \mu^4\Big)\Big\} e^{-\frac{\theta_o \mu^2}{2}}\Big] +$
bkθot μ+ *Tnsdndr*− μ+*θo 6 T3nsdndr3*− μ*3e*− *θo*μ*22*− *1θot ak2T2nsdndr2+bkTnsdndr Hrk*

$+\ a_k \Big[\frac{\mu^2}{2} + \Big\{\frac{1}{2}\Big(\frac{T^2}{(n_{sd} n_{dr})^2} - \mu^2\Big) + \frac{\theta_o}{8}\Big(\frac{T^4}{(n_{sd} n_{dr})^4} - \mu^4\Big)\Big\} e^{-\frac{\theta_o \mu^2}{2}}\Big] + b_k \Big[\mu + \Big\{\Big(\frac{T}{n_{sd} n_{dr}} - \mu\Big) + \frac{\theta_o}{6}\Big(\frac{T^3}{(n_{sd} n_{dr})^3} -$
μ*3e*− *θo*μ*22- ak T2nsdndr2+bkTnsdndr]Drk*+(a+bQr)] (14)

(b) Cost Structure of Distributor

We consider one distributor in the supply chain. Let I_d (t) be the inventory level at the distributor at any time t .The demand rate at the distributor is give by $\sum_{k=1}^{3}(a_k t + b_k)$. The deterioration rate of the inventory is $\theta_o t$. The inventory system at the distributor is shown in figure-2.

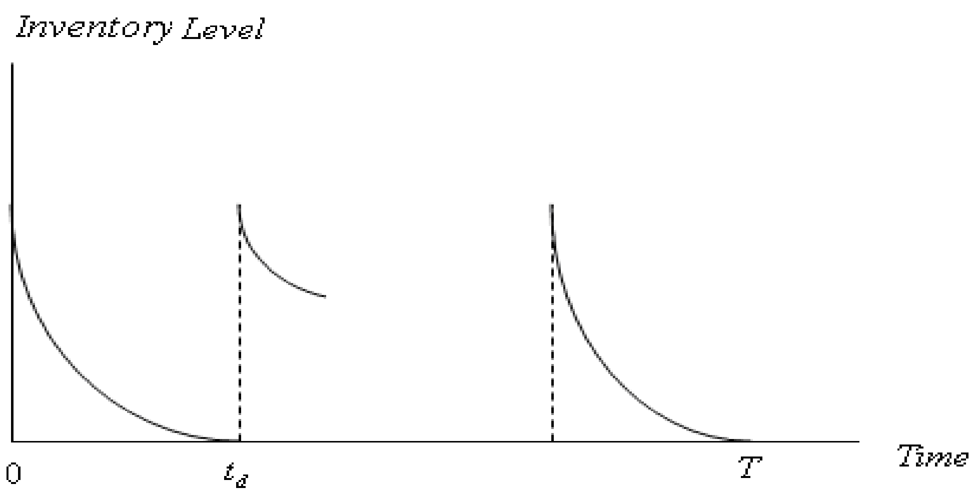

Figure-2

The differential equation governing the inventory at the distributor is given by

$$\frac{dI_d(t)}{dt} = -\sum_{k=1}^{3}(a_k t + b_k), \qquad 0 \le t \le \mu \qquad (15)$$
$$\frac{dI_d(t)}{dt} = -\sum_{k=1}^{3}(a_k t + b_k) - \theta_o t\, I_{rk}, \qquad \mu \le t \le t_d \qquad (16)$$

The various boundary conditions are given by

$$I_d(0) = S_d\ ,\ I_d(t_d) = 0 \qquad (17)$$

The solution of equation (15) is given by

$$I_d\ (t) = - \sum_{k=1}^{3}\left(\frac{a_k\, t^2}{2} + b_k t\right) + c_3 \quad (18)$$

Using boundary condition (17) in equation (18), we get

$$c_3 = S_d$$

Now substituting the value of c_3 in equation (18) , we get

$$I_d\ (t) = - \sum_{k=1}^{3}\left(\frac{a_k\, t^2}{2} + b_k t\right) + S_d \quad , \quad 0 \le t \le \mu \quad (19)$$

The solution of equation (16) is given by

$$I_d(t) = -\sum_{k=1}^{3} \left(a_k \frac{t^2}{2} + b_k t + a_k \frac{t^4\theta_o}{8} + b_k \frac{t^3\theta_o}{6}\right) e^{-\frac{\theta_o t^2}{2}} + c_4 e^{-\frac{\theta_o t^2}{2}} \quad (20)$$

Using boundary condition (17) in equation (20) , we get

$$I_d(t) = \sum_{k=1}^{3} \left[\frac{a_k}{2} ({t^2}_d - t^2) + b_k(t_d - t) + \frac{a_k\theta_o}{8} ({t^4}_d - t^4) + \frac{b_k\theta_o}{6} ({t^3}_d - t^3)\right] e^{-\frac{\theta_o t^2}{2}} \quad (21)$$

Where, $\mu \le t \le t_d$

From (19) and (21) , we have

$$- \sum_{k=1}^{3}\left(\frac{a_k\, \mu^2}{2} + b_k \mu\right) + S_d =$$
$$\sum_{k=1}^{3} \left[\frac{a_k}{2} ({t^2}_d - \mu^2) + b_k(t_d - \mu) + \frac{a_k\theta_o}{8} ({t^4}_d - \mu^4) + \frac{b_k\theta_o}{6} ({t^3}_d - \mu^3)\right] e^{-\frac{\theta_o \mu^2}{2}}$$

$$\therefore S_d = \sum_{k=1}^{3}\left(\frac{a_k\, \mu^2}{2} + b_k \mu\right) +$$
$$\sum_{k=1}^{3} \left[\frac{a_k}{2} ({t^2}_d - \mu^2) + b_k(t_d - \mu) + \frac{a_k\theta_o}{8} ({t^4}_d - \mu^4) + \frac{b_k\theta_o}{6} ({t^3}_d - \mu^3)\right] e^{-\frac{\theta_o \mu^2}{2}} \quad (22)$$

The deteriorating quantities per replenishment cycle can be obtained as

$$R_d = S_d - \sum_{k=1}^{3} \int_0^{t_d} (a_k t + b_k)\, dt$$
$$= S_d - \sum_{k=1}^{3} \left(a_k \frac{{t^2}_d}{2} + b_k t_d\right)$$
$$= \sum_{k=1}^{3}\left(\frac{a_k\, t^2}{2} + b_k t\right) +$$
$$\sum_{k=1}^{3} \left[\frac{a_k}{2} ({t^2}_d - t^2) + b_k(t_d - t) + \frac{a_k\theta_o}{8} ({t^4}_d - t^4) + \frac{b_k\theta_o}{6} ({t^3}_d - t^3)\right] e^{-\frac{\theta_o t^2}{2}} -$$
$$\sum_{k=1}^{3} \left(a_k \frac{{t^2}_d}{2} + b_k t_d\right) \quad (23)$$

The actual deteriorating quantities per replenishment cycle can be obtained as

$$R_d = \sum_{k=1}^{3}\left(\frac{a_k\, \mu^2}{2} + b_k \mu\right) +$$
$$\sum_{k=1}^{3} \left[\frac{a_k}{2} ({t^2}_d - \mu^2) + b_k(t_d - \mu) + \frac{a_k\theta_o}{8} ({t^4}_d - \mu^4) + \frac{b_k\theta_o}{6} ({t^3}_d - \mu^3)\right] e^{-\frac{\theta_o \mu^2}{2}} -$$
$$\sum_{k=1}^{3} \left(a_k \frac{{t^2}_d}{2} + b_k t_d\right) - \sum_{k=1}^{3} R_{rk} = A_d \theta_o t \quad (24)$$

The deterioration cost per replenishment cycle is given by

$$DC_d = R_d \times D_d$$
$$= [\sum_{k=1}^{3}\left(\frac{a_k\, \mu^2}{2} + b_k \mu\right) +$$
$$\sum_{k=1}^{3} \left[\frac{a_k}{2} ({t^2}_d - \mu^2) + b_k(t_d - \mu) + \frac{a_k\theta_o}{8} ({t^4}_d - \mu^4) + \frac{b_k\theta_o}{6} ({t^3}_d - \mu^3)\right] e^{-\frac{\theta_o \mu^2}{2}} -$$
$$\sum_{k=1}^{3} \left(a_k \frac{{t^2}_d}{2} + b_k t_d\right) - \sum_{k=1}^{3} R_{rk} \ ]\ D_d \quad (25)$$

From equation (24), we have

$$A_d = \frac{1}{\theta_o t} [\sum_{k=1}^{3}\left(\frac{a_k\, \mu^2}{2} + b_k \mu\right)$$

$+\sum_{k=1}^{3}\left[\frac{a_k}{2}\left(t^2{}_d-\mu^2\right)+b_k\left(t_d-\mu\right)+\frac{a_k\theta_o}{8}\left(t^4{}_d-\mu^4\right)+\frac{b_k\theta_o}{6}\left(t^3{}_d-\mu^3\right)\right]e^{-\frac{\theta_o\mu^2}{2}}]$

$-\frac{1}{\theta_o t}\sum_{k=1}^{3} R_{rk}$ (26)

The carrying cost per replenishment cycle is given by

$HC_d=\frac{H_d}{\theta_o t}[\sum_{k=1}^{3}\left(\frac{a_k\mu^2}{2}+b_k\mu\right)$

$+\sum_{k=1}^{3}\left[\frac{a_k}{2}\left(t^2{}_d-\mu^2\right)+b_k\left(t_d-\mu\right)+\frac{a_k\theta_o}{8}\left(t^4{}_d-\mu^4\right)+\frac{b_k\theta_o}{6}\left(t^3{}_d-\mu^3\right)\right]e^{-\frac{\theta_o\mu^2}{2}}-\sum_{k=1}^{3} R_{rk}]$

By summing the ordering cost, purchase cost, the carrying cost and deteriorating cost, the transportation cost, the distributor's total cost during time T unit can be expressed as

$TC_d(T)=C_d n_{sd}+HC_d n_{sd}+DC_d n_{sd}+a+bQ_r$ (27)

(c) Cost structure of N-suppliers

We consider the number of suppliers N equal to three in the supply chain. Let $I_{st}(t)$ be the inventory level at the suppliers at any time t. The demand rate at the supplier is give by$\sum_{k=1}^{3} m_i(a_k t+b_k)$, where m_i is the proportion of order arrangement for supplier i, i=1, 2. The deterioration rate of the inventory is $\theta_o t$ .The inventory system is shown in figure-3 given below:

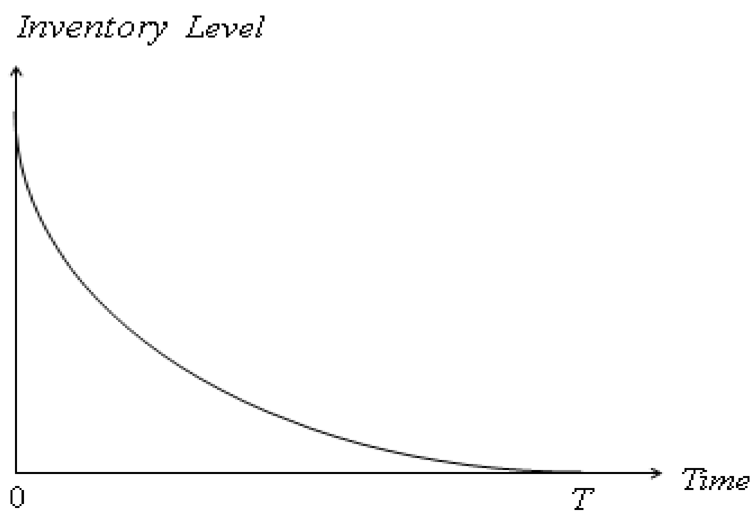

Figure-3

The differential equation governing the inventory at the suppliers is given

$\frac{dI_{si}(t)}{dt}=-\sum_{k=1}^{3} m_i(a_k t+b_k)$, $0\le t\le\mu$ (28)

$\frac{dI_{si}(t)}{dt}=-\sum_{k=1}^{3} m_i(a_k t+b_k)-\theta_o t\, I_{pi}$, $\mu\le t\le T$ (29)

The various boundary conditions are given by

$I_{si}(0)=S_{si}$, $I_{si}(T)=0$ (30)

The solution of equation (28) is given by

$I_d(t)=-\sum_{k=1}^{3} m_i\left(\frac{a_k t^2}{2}+b_k t\right)+c_5$ (31)

Using boundary condition (30) in equation (31), we get

$c_5=S_{si}$

Now substituting the value of c_5 in equation (31) , we get

$I_{si}(t)=-\sum_{k=1}^{3} m_i\left(\frac{a_k t^2}{2}+b_k t\right)+S_{si}$, $0\le t\le\mu$ (32)

The solution of equation (29) is given by

$$I_{si}(t) = -\sum_{k=1}^{3} m_i \left(a_k \frac{t^2}{2} + b_k t + a_k \frac{t^4\theta_o}{8} + b_k \frac{t^3\theta_o}{6}\right) e^{-\frac{\theta_o t^2}{2}} + c_6 e^{-\frac{\theta_o t^2}{2}} \quad (33)$$

Using boundary condition (30) in equation (33) , we get

$$I_{si}(t) = \sum_{k=1}^{3} m_i \left[\frac{a_k}{2}(T^2 - t^2) + b_k(T - t) + \frac{a_k\theta_o}{8}(T^4 - t^4) + \frac{b_k\theta_o}{6}(T^3 - t^3)\right] e^{-\frac{\theta_o t^2}{2}} \quad (34)$$

Where $\mu \le t \le T$

From (32) and (34), we have

$$-\sum_{k=1}^{3} m_i \left(\frac{a_k \mu^2}{2} + b_k \mu\right) + S_{si} = \sum_{k=1}^{3} m_i \left[\frac{a_k}{2}(T^2 - \mu^2) + b_k(T - \mu) + \frac{a_k\theta_o}{8}(T^4 - \mu^4) + \frac{b_k\theta_o}{6}(T^3 - \mu^3)\right] e^{-\frac{\theta_o \mu^2}{2}}$$

$$\therefore S_{si} = \sum_{k=1}^{3} m_i \left(\frac{a_k \mu^2}{2} + b_k \mu\right) + \sum_{k=1}^{3} m_i \left[\frac{a_k}{2}(T^2 - \mu^2) + b_k(T - \mu) + \frac{a_k\theta_o}{8}(T^4 - \mu^4) + \frac{b_k\theta_o}{6}(T^3 - \mu^3)\right] e^{-\frac{\theta_o \mu^2}{2}} \quad (35)$$

The actual deteriorating quantities at supplier i during supply cycle is given by

$$R_{si} = S_{si} - \sum_{k=1}^{3} \int_0^T (a_k t + b_k)\, dt - \sum_{k=1}^{3} m_i R_d$$

$$= S_{si} - \sum_{k=1}^{3} \left(\frac{a_k T^2}{2} + b_k T\right) - \sum_{k=1}^{3} m_i R_d$$

$$= \sum_{k=1}^{3} m_i \left(\frac{a_k \mu^2}{2} + b_k \mu\right) + \sum_{k=1}^{3} m_i \left[\frac{a_k}{2}(T^2 - \mu^2) + b_k(T - \mu) + \frac{a_k\theta_o}{8}(T^4 - \mu^4) + \frac{b_k\theta_o}{6}(T^3 - \mu^3)\right] e^{-\frac{\theta_o \mu^2}{2}} - \sum_{k=1}^{3} \left(\frac{a_k T^2}{2} + b_k T\right) - \sum_{k=1}^{3} m_i R_d = A_{si}\theta_o t \quad (36)$$

Therefore the deterioration cost at supplier during supply cycle is given by

$$DC_{si} = R_{si} \times D_{si}$$

$$= \left[\sum_{k=1}^{3} m_i \left(\frac{a_k \mu^2}{2} + b_k \mu\right) + \sum_{k=1}^{3} m_i \left[\frac{a_k}{2}(T^2 - \mu^2) + b_k(T - \mu) + \frac{a_k\theta_o}{8}(T^4 - \mu^4) + \frac{b_k\theta_o}{6}(T^3 - \mu^3)\right] e^{-\frac{\theta_o \mu^2}{2}} - \sum_{k=1}^{3} \left(\frac{a_k T^2}{2} + b_k T\right) - \sum_{k=1}^{3} m_i R_d \right] D_{si} \quad (37)$$

From equation (36) and (37) the inventory cost at the suppliers during the cycle can be obtained as

$$HC_{si} = \sum_{i=1}^{2}\sum_{k=1}^{3} \frac{1}{\theta_o t}\left[S_{si} - \sum_{k=1}^{3} \left(\frac{a_k T^2}{2} + b_k T\right) - \sum_{k=1}^{3} m_i R_d\right] H_{si} \quad (38)$$

By summing the ordering cost, the purchasing cost, the carrying cost, the retailer's cost total cost per replenishment cycle can be expressed as

$$TC_{si}(T) = \sum_{i=1}^{2}(C_{si} + HC_{si} + D_{si})\, n_{sd}$$

$$= \sum_{i=1}^{2} C_{si} n_{sd} + \sum_{i=1}^{2}\sum_{k=1}^{3} \frac{1}{\theta_o t}\left[S_{si} - \sum_{k=1}^{3} \left(\frac{a_k T^2}{2} + b_k T\right) - \sum_{k=1}^{3} m_i R_d\right] H_{si} n_{sd} + \left[S_{si} - \sum_{k=1}^{3} \left(\frac{a_k T^2}{2} + b_k T\right) - \sum_{k=1}^{3} m_i R_d\right] D_{si} n_{sd} \quad (39)$$

Where , $S_{si} = \sum_{k=1}^{3} m_i \left(\frac{a_k \mu^2}{2} + b_k \mu\right) + \sum_{k=1}^{3} m_i \left[\frac{a_k}{2}(T^2 - \mu^2) + b_k(T - \mu) + \frac{a_k\theta_o}{8}(T^4 - \mu^4) + \frac{b_k\theta_o}{6}(T^3 - \mu^3)\right] e^{-\frac{\theta_o \mu^2}{2}}$

(d) Joint Total Cost Structure of N-suppliers-One-distributor-N-retailer

In this model, three suppliers, one distributor and three retailers are considered. Thus there are three players (suppliers, distributor and retailers) in the serial inventory of multi-echelon supply chain. The integrated total cost can be expressed as

$TC = TC_r + TC_d + TC_s$

The optimization problem is a constrained nonlinear program given by

Minimize $TC = TC_r + TC_d + TC_s$, subject to condition $t_{rk} = \frac{T}{n_{sd}\, n_{dr}}$, $n_{ik} \in N$,

where, k= 1, 2, 3 and i= 1, 2

The variables of the integrated total cost are T and n_{ik} , i= 1, 2 and k= 1, 2, 3.

For the minimization of the integrated total cost TC, these variables must satisfy the following conditions

$$\frac{dTC}{dT} = 0 \quad (40)$$

and $TC(n_{ik}^* - 1) \geq TC(n_{ik}^*) \geq TC(n_{ik}^* + 1)$ (41)

where k= 1, 2, 3 and i= 1, 2

The optimal values of T and n_{ik} are denoted by T^* and n_{ik}^* respectively and these values have to satisfy equations (40) and (41). Also n_{ik}^* is an integer in vicinity of $n_{ik}^\#$ that satisfy the condition

$\frac{dTC}{dn_{ik}^\#} = 0$, where k= 1, 2, 3 and i= 1, 2 (42)

CONCLUSIONS:

In this paper, an integrated supply chain model has been developed with linearly varying demand rate and variable rate of deterioration. The replenishment of items at the distributor and the retailer is considered as instantaneous. Shortage of the items is not allowed. Cost minimization technique has been used to obtain the optimal values of parameter. This work can further be extended for other form of demand rate, deterioration rate and for the case of shortage

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